

Generation of powerful terahertz emission in a beam-driven strong plasma turbulence

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Abstract.

Generation of terahertz electromagnetic radiation due to coalescence of upper-hybrid waves in the long-wavelength region of strong plasma turbulence driven by a high-current relativistic electron beam in a magnetized plasma is investigated. The width of frequency spectrum as well as angular characteristics of this radiation for various values of plasma density and turbulence energy are calculated using the simple theoretical model adequately describing beam-plasma experiments at mirror traps. It is shown that the power density of electromagnetic emission at the second harmonic of plasma frequency in the terahertz range for these laboratory experiments can reach the level of 1 MW/cm^3 with 1% conversion efficiency of beam energy losses to electromagnetic emission.

PACS numbers: 52.25.Os, 52.35.Ra, 52.50.Gj

1. Introduction

It is well known that a turbulent plasma is a source of electromagnetic emission at the fundamental plasma frequency ω_p and its second harmonic. These emission mechanisms have been found to play an important role in different space phenomena such as type III solar radio bursts [1, 2, 3, 4, 5] and radiations of planet's magnetospheres [6, 7]. Plasma emissions at ω_p and $2\omega_p$ have been also observed in laboratory beam-plasma experiments [8, 9, 10, 11]. In contrast to the problem of type III radio bursts, in which the density of wave energy W is saturated at weakly turbulent levels $W/nT \sim 10^{-5}$ (n is the plasma density, T is the temperature of plasma electrons), we focus our attention on the regime of strong plasma turbulence $W/nT \sim 10^{-2} - 10^{-1}$, which is more appropriate for laboratory experiments with powerful electron beams [12, 13, 14].

Beam-plasma experiments at mirror traps [14, 15] have demonstrated that the relativistic electron beam with the typical energy 1 MeV and current density 1 kA/cm^2 injected into the plasma with $n = 2 \cdot 10^{14} \text{ cm}^{-3}$ loses about 30% of its energy over the length of 1 m. It means that the averaged power density pumping by the beam to the plasma turbulence can be estimated as $\sim 10 \text{ MW/cm}^3$. The rate of beam energy losses, however, is not uniform over this length. The estimate for the peak pumping power can be derived assuming that a significant part of beam energy goes to excitation of the large amplitude coherent wave-packet at the entrance to the plasma column. Since the beam is decelerated over the packet length $l \sim v_b/\Gamma$ by $\Delta v \sim v_b\Gamma/\omega_p$ [16], where Γ is

the growth-rate of the two-stream instability and v_b is the beam velocity, the maximum power of beam energy losses reaches the value $\sim 100 \text{ MW/cm}^3$. Thus, if only 1% of pumping power is able to convert to electromagnetic radiation, the maximum power density of plasma emission produced in beam-plasma experiments at mirror traps can reach the range of 1 MW/cm^3 . The same level of conversion has been observed recently in laboratory experiments [11] with the low plasma density $n = 2 \cdot 10^{14} \text{ cm}^{-3}$, that is why it allows us to suppose that, in a denser plasma, a beam excited turbulence can be also considered as an efficient source of powerful electromagnetic radiation. The aim of this paper is to calculate the conversion efficiency as well as absolute values of emission power for the case when second harmonic plasma emission falls in the terahertz frequency range.

Our calculations of the spectral power of electromagnetic emission produced in a turbulent magnetized plasma are based on the theoretical model [17], in which it is assumed that electromagnetic waves are generated predominantly in the source region of strong plasma turbulence due to coalescence of upper-hybrid waves. Since theoretical predictions have been found to agree with recent experimental results [11] obtained at the GOL-3 multimirror trap in the low density regime, we can use this model to study spectral and angular characteristics of electromagnetic emission for the whole ranges of plasma density and turbulence energy, which can be achieved in our beam-plasma experiments.

Thus, in Section 2, we formulate the main ideas of theoretical model that takes into account not only spontaneous generation of electromagnetic waves in coalescence processes, but also contributions of induced inverse processes. In Section 3, we present calculations of second harmonic emission power in the terahertz frequency range and study whether this emission can escape from the plasma. Our main results are summarized in concluding Section 4.

2. Theoretical model

To calculate the emission power from turbulent magnetized plasma we use the model of strong plasma turbulence proposed in [17]. According to this model, we assume that most of wave energy is concentrated in upper-hybrid modes, which occupy the long-wavelength region of turbulent spectrum with wavenumbers $k < \sqrt{W/nT}/r_D$ (r_D is the Debye length). The saturation level of this energy is determined by the balance between the constant power P_b pumping to the turbulence by the beam and the power dissipated due to the wave collapse. It results in the following relation between the pumping power and the turbulence energy:

$$P_b \sim \omega_p n T \left(\frac{m_e}{m_i} \right)^{1/2} \left(\frac{W}{nT} \right)^2, \quad (1)$$

where m_i is the ion mass and m_e is the electron mass.

We calculate the electromagnetic emission power P and the conversion efficiency $\epsilon = P/P_b$ assuming that electromagnetic waves are generated due to coalescence of long-wavelength upper-hybrid waves. Since in the source region of turbulent spectrum these waves are not trapped in collapsing caverns, nonlinear interaction between electromagnetic and upper-hybrid modes can be treated in the framework of weak turbulence. In this case, generation of electromagnetic radiation is described by the

equation

$$\frac{\partial W_k^t}{\partial t} = P_k - 2\gamma_k W_k^t, \quad (2)$$

where P_k is the power of spontaneous emission produced in the coalescence process $\ell + \ell \rightarrow t$ and γ_k is the nonlinear dissipation rate due to the decay $t \rightarrow \ell + \ell$. In dimensionless units $\omega_p t$, ω/ω_p , $x\omega_p/c$, kc/ω_p , for time, frequency, position and wavenumber, respectively, the spectral wave energy is normalized by the condition

$$\int W_k^\sigma d^3k = \frac{W^\sigma}{nm_e c^2}, \quad (3)$$

where c is the speed of light. In the cold plasma limit, dimensionless values of P_k and γ_k take the forms

$$P_k = \frac{2\pi}{\omega_k^t (\partial\Lambda/\partial\omega)_{\omega_k^t}} \int \frac{W_{k_1}^\ell W_{k_2}^\ell |G_{k,k_1,k_2}^{t\ell\ell}|^2 \Delta_{k,k_1,k_2}}{\omega_{k_1}^\ell (\partial\Lambda/\partial\omega)_{\omega_{k_1}^\ell} \omega_{k_2}^\ell (\partial\Lambda/\partial\omega)_{\omega_{k_2}^\ell}} \times \delta(\mathbf{k} - \mathbf{k}_1 - \mathbf{k}_2) d^3k_1 d^3k_2, \quad (4)$$

$$\gamma_k = \frac{1}{\omega_k^t (\partial\Lambda/\partial\omega)_{\omega_k^t}} \int \frac{W_{k_2}^\ell \delta(\mathbf{k} - \mathbf{k}_1 - \mathbf{k}_2)}{\omega_{k_1}^\ell (\partial\Lambda/\partial\omega)_{\omega_{k_1}^\ell} \omega_{k_2}^\ell (\partial\Lambda/\partial\omega)_{\omega_{k_2}^\ell}} \times \left[\frac{-iG_{k,k_1,k_2}^{t\ell\ell} G_{k_1,-k_2,k}^{\ell\ell t}}{\omega_{k_1}^\ell + \omega_{k_2}^\ell - \omega_k^t - i\nu} + \text{c. c.} \right] d^3k_1 d^3k_2, \quad (5)$$

where we introduce the following notations:

$$\Lambda^\sigma(\mathbf{k}, \omega) = |\mathbf{k} \cdot \mathbf{e}_k^\sigma|^2 - k^2 + \omega^2 (\mathbf{e}_k^{*\sigma} \hat{\varepsilon}_k^\sigma \mathbf{e}_k^\sigma), \quad (6)$$

$$G_{k,k_1,k_2}^{\sigma\sigma'\sigma''} = \frac{\omega_+}{\omega_{k_1}^{\sigma'}} \left(\mathbf{e}_k^{*\sigma} \hat{T}_{k_2}^{\sigma''} \mathbf{e}_{k_2}^{\sigma''} \right) \left(\mathbf{k}_1 \hat{T}_{k_1}^{\sigma'} \mathbf{e}_{k_1}^{\sigma'} \right) + \frac{\omega_+}{\omega_{k_2}^{\sigma''}} \left(\mathbf{e}_k^{*\sigma} \hat{T}_{k_1}^{\sigma'} \mathbf{e}_{k_1}^{\sigma'} \right) \left(\mathbf{k}_2 \hat{T}_{k_2}^{\sigma''} \mathbf{e}_{k_2}^{\sigma''} \right) + \mathbf{e}_k^{*\sigma} \hat{T}^+ \mathbf{g}, \quad (7)$$

$$\mathbf{g} = \left(\mathbf{k}_2 \hat{T}_{k_1}^{\sigma'} \mathbf{e}_{k_1}^{\sigma'} \right) \left[\hat{T}_{k_2}^{\sigma''} \cdot \mathbf{e}_{k_2}^{\sigma''} - \left(1 - \frac{\Omega^2}{(\omega_{k_2}^{\sigma''})^2} \right) \mathbf{e}_{k_2}^{\sigma''} \right] + \mathbf{k}_2 \left(\mathbf{e}_{k_2}^{\sigma''} \hat{T}_{k_1}^{\sigma'} \mathbf{e}_{k_1}^{\sigma'} \right) + (k_1, \sigma' \rightleftharpoons k_2, \sigma''), \quad (8)$$

$$\hat{T}_k^\sigma = (\omega_k^\sigma)^2 \left(\hat{I} - \hat{\varepsilon}_k^\sigma \right), \quad \omega_+ = \omega_{k_1}^{\sigma'} + \omega_{k_2}^{\sigma''}. \quad (9)$$

Here, eigenfrequencies ω_k^σ and eigenvectors \mathbf{e}_k^σ of linear plasma modes are determined by the standard dispersion equation with the dielectric tensor $\hat{\varepsilon}_k^\sigma$ (\hat{I} is the unit matrix). In contrast to the similar calculations of second harmonic emission [18, 19] based on the standard weak turbulence theory, we take into account model damping of two-time correlation functions, which is used to describe the effect of finite life-time of upper-hybrid plasmons due to their scattering off density fluctuations with the typical frequency $\nu = \omega_p W^\ell/(nT)$. It results in correlation broadening of the resonance $\omega_k^t - \omega_{k_1}^\ell - \omega_{k_2}^\ell = 0$, which is described in (4) by the function

$$\Delta_{k,k_1,k_2} = \frac{2\nu/\pi}{(\omega_k^t - \omega_{k_1}^\ell - \omega_{k_2}^\ell)^2 + 4\nu^2}. \quad (10)$$

This function shows that the width of frequency spectrum of second harmonic electromagnetic emission depends essentially on nonlinear effects as well as on thermal and magnetic corrections to the linear dispersion of upper-hybrid modes. In order to minimize the frequency width of radiation, we should consider the case, when the ratio of the electron cyclotron frequency to the plasma frequency becomes low $\Omega = \omega_c/\omega_p = 0.2$ resulting in comparable contributions of magnetic and thermal effects for the typical plasma temperature $T = 1$ keV in mirror traps. As to account for the effect of finite temperature, we modify the eigenfrequencies and eigenvectors of linear plasma modes, but we neglect modifications in the nonlinear current $G_{k,k_1,k_2}^{t\ell\ell}$, which is much less sensitive to thermal corrections than Δ_{k,k_1,k_2} . Thus, to calculate ω_k^σ , we use the fluid approximation and solve the linear dispersion relation with the dielectric tensor:

$$\varepsilon_{xx} = 1 - A \left(1 - \frac{k_{\parallel}^2 V_T^2}{\omega^2} \right), \quad (11)$$

$$\varepsilon_{xy} = -\varepsilon_{yx} = i \frac{\Omega}{\omega} A \left(1 - \frac{k_{\parallel}^2 V_T^2}{\omega^2} \right), \quad (12)$$

$$\varepsilon_{yy} = 1 - A \left(1 - \frac{k_{\perp}^2 V_T^2}{\omega^2} \right), \quad (13)$$

$$\varepsilon_{xz} = \varepsilon_{zx} = -A \frac{k_{\parallel} k_{\perp} V_T^2}{\omega^2}, \quad (14)$$

$$\varepsilon_{yz} = -\varepsilon_{zy} = -i \frac{\Omega}{\omega} A \frac{k_{\parallel} k_{\perp} V_T^2}{\omega^2}, \quad (15)$$

$$\varepsilon_{zz} = 1 - A \left(1 - \frac{k_{\perp}^2 V_T^2 + \Omega^2}{\omega^2} \right), \quad (16)$$

$$A = \left(\omega^2 - \Omega^2 - k^2 V_T^2 + \frac{\Omega^2}{\omega^2} k_{\parallel}^2 V_T^2 \right)^{-1},$$

where $V_T^2 = 3T/(m_e c^2)$, magnetic field is directed along z -axis and $\mathbf{k} = (k_{\perp}, 0, k_{\parallel})$ is the wave vector with the length $k = (k_{\perp}^2 + k_{\parallel}^2)^{1/2}$.

If the path lengths of spontaneously generated electromagnetic waves are larger than the typical size of confined plasma $l_k = v_g/\gamma_k > L$ (v_g is the group velocity of electromagnetic wave), decay processes $t \rightarrow \ell + \ell$ do not play a role, and the second term in (2) can be omitted. Thus, in the case of azimuthally symmetric turbulence, the spectral emission power in units of $nm_e c^2$ is given by the integral

$$\frac{dP}{d\omega} = 2\pi \int_0^\pi \sin \theta d\theta \left(\frac{k^2}{d\omega/dk} P_k \right)_{k(\omega)}, \quad (17)$$

where $k(\omega)$ is the solution of $\omega = \omega_k^t$ and θ is the polar angle of \mathbf{k} .

3. Computation results

Let us study the spectral and angular characteristics of second harmonic plasma emission for various regimes of beam-plasma interaction, which can be realized in mirror traps. We will vary the plasma density from $2 \cdot 10^{14} \text{ cm}^{-3}$ to $5 \cdot 10^{15} \text{ cm}^{-3}$ for different fixed values of turbulence energy $W/nT = 0.01, 0.05, 0.1$ and for the fixed parameters $T = 1$ keV and $\Omega = 0.2$.

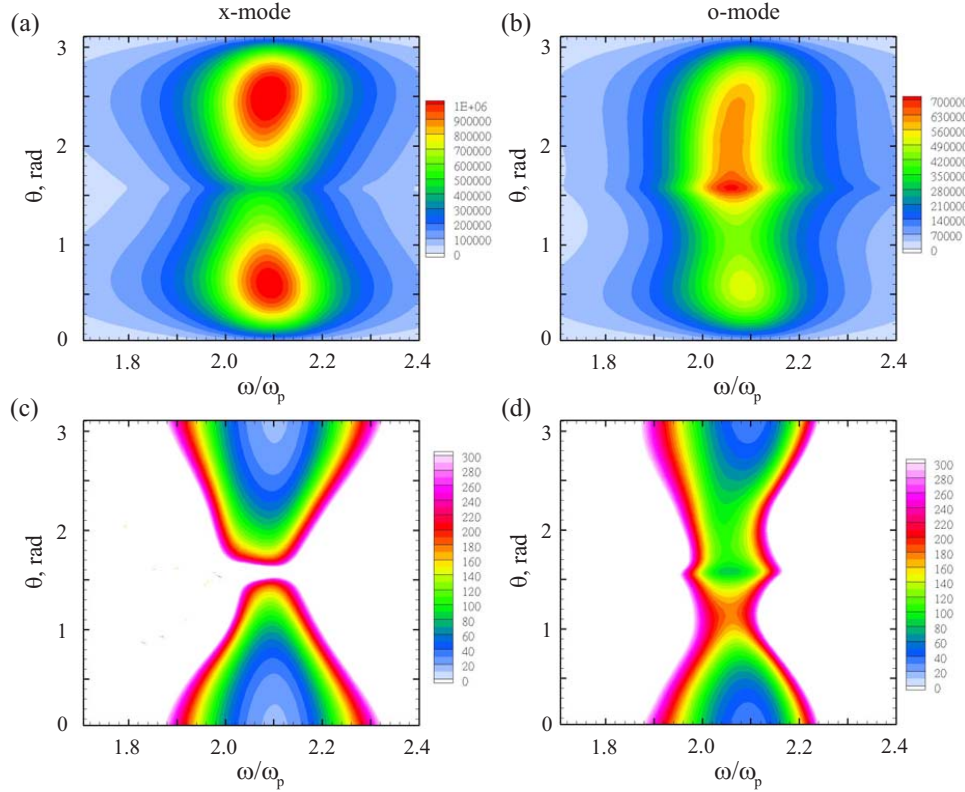


Figure 1. The spectral power $dP/(d\omega d\theta)$ (in W/cm^3) for x-mode emission (a) and o-mode emission (b). The path length (in cm) for x-modes (c) and o-modes (d).

We assume here that the source region of turbulent spectrum contains the anisotropic population of resonant waves, which are directly pumped by the beam, and the isotropic population of nonresonant waves, which are the products of beam-driven waves scattering off density fluctuations. In our computations, the isotropic part occupies uniformly the spectral region $kc/\omega_p \in (0.1, k_m c/\omega_p)$, where the upper bound corresponds to the typical wavenumber of the modulation instability $k_m \simeq \sqrt{W/nT}/r_D$ and the lower bound excludes modes with wavelengths larger than the typical plasma size. According to the model [17], the anisotropic population of beam-driven waves contains a small fraction of energy (10%) and occupies the region: $kc/\omega_p \in (1.1, 1.3)$ and $\theta \in (0, 0.3)$.

In the regime with $n = 3 \cdot 10^{15} \text{ cm}^{-3}$ and $W/nT = 0.05$, computation results for the spectral density of emission power $dP/(d\omega d\theta)$ for extraordinary (x) and ordinary (o) electromagnetic modes are presented in Figures 1(a) and 1(b), respectively. It is seen that x -mode emission is dominated by the contribution of obliquely propagating waves with $\theta = 30^\circ$ and $\theta = 150^\circ$, whereas the most intensive o -mode emission is concentrated near the transverse to the magnetic field direction. Calculations of the path length l_k for these emissions are shown in Figures 1(c) and 1(d). As one can see, the path length reaches the minimal value 20-40 cm for longitudinally

propagating modes regardless of their polarization. For transverse propagation, this length increases up to 90 cm for the o -mode and exceeds 3 m for the x -mode. It means that emissions of both electromagnetic modes, generated in the plasma column with the diameter 5–6 cm typical to our beam-plasma experiments, are able to escape from the plasma and can be used for different applications.

Let us now find out how the frequency spectrum of electromagnetic emission, described by $dP/d\omega$, depends on the plasma density. As a function of dimensionless

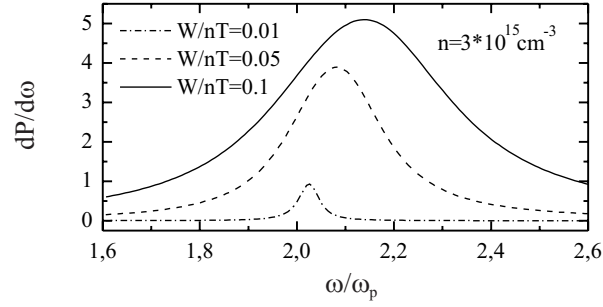


Figure 2. The spectral power of second harmonic emission $dP/d\omega$ in MW/cm^3 for different values of turbulence energy W/nT .

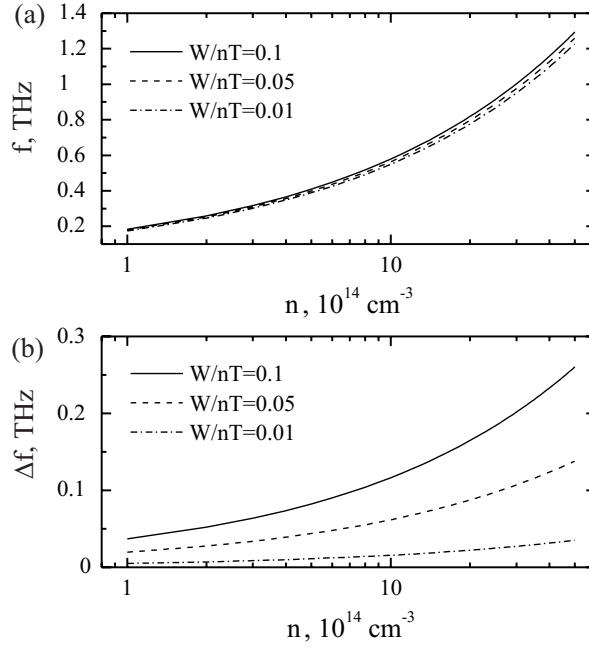


Figure 3. Dependences of f (a) and Δf (b) on the plasma density for different values of turbulence energy W/nT .

frequency ω/ω_p , the form of this spectrum is affected by the level of turbulence only. For the density $n = 3 \cdot 10^{15} \text{ cm}^{-3}$ and different values of turbulence energy, examples

of frequency spectra, derived by summation over polarizations, are shown in Figure 2. In dimensional form, the linear frequency f corresponding to the maximum of the spectral power $dP/d\omega$ and the width at half maximum Δf for a fixed turbulence energy depend on the plasma density as follows: $f, \Delta f \propto n^{1/2}$. Figures 3(a) and 3(b) demonstrate that f and especially Δf grow with the increase of the turbulence level. It means that the spectral width of plasma emission in regimes of rather strong plasma turbulence is mainly determined by correlation broadening of resonances in nonlinear three-wave interactions.

To compute the integral power of second harmonic emission from the unit volume of turbulent plasma, we integrate the function $dP/d\omega$ over the frequency range $\omega/\omega_p \in (1.6, 2.6)$ and sum over polarizations. It is seen from Figure 4, that this power

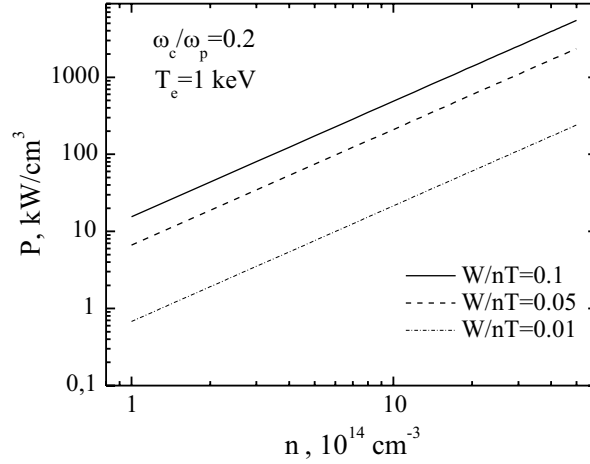


Figure 4. The dependence of integral second harmonic emission power on the plasma density for different values of turbulence energy W/nT .

increases like $P \propto n^{3/2}$ with the increase of plasma density for the fixed turbulence energy and reaches the value 1 MW/cm³ in the regime with $n = 3 \cdot 10^{15} \text{ cm}^{-3}$ and $W/nT = 0.05$. It is interesting to note that these calculations adequately describes recent experimental results obtained in the strong magnetic field $\Omega = 0.8$ [11]. Indeed, the 1 kW/cm³ range of emission power, observed experimentally in the regime with $n = 2 \cdot 10^{14} \text{ cm}^{-3}$ and $W/nT = 0.01$, is also reproduced by calculations presented in Figure 4. From comparison with the results of [17] we can conclude that the external magnetic field broadens the spectral width of electromagnetic emission, but does not change drastically the integral power. Since the pumping power (1) has the same dependence on plasma density as the emission power, the conversion efficiency ϵ in our model is completely determined by the turbulence energy. For $W/nT = 0.05$, this efficiency is estimated as $\epsilon \simeq 1\%$.

To make clearer understanding how computation results are sensitive to the shape of turbulent spectrum, we calculate the spectral emission power for different spectra corresponding to the same turbulence energy (Figure 5). When we replace the uniform isotropic spectrum of upper-hybrid modes used in previous calculations on more realistic falling spectra, changes in the integral emission power do not exceed 20%.

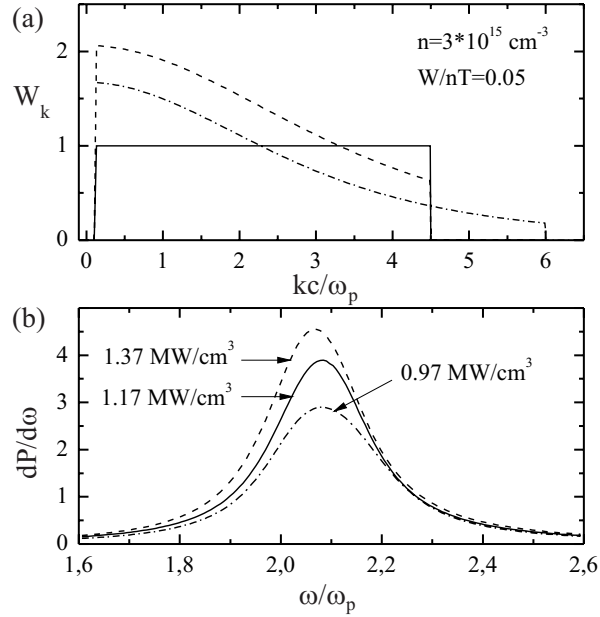


Figure 5. Different shapes of the isotropic part of turbulent k -spectra (a) and the corresponding spectral power of electromagnetic emission (b).

It should be emphasized that the high value of turbulence energy $W/nT = 0.05$, which is required to generate 1 THz emission with the power density 1 MW/cm^3 , can be achieved in beam-plasma experiments at the GOL-3 multimirror trap. Indeed, for the typical beam energy 1 MeV and current density 1.5 kA/cm^2 , the pumping power reaches the required level of 100 MW/cm^3 inside the region of most intensive beam-plasma interaction, where large amplitude coherent wave-packets are excited. Since the length of this region is estimated as $l \sim v_b/\Gamma \simeq 1 - 3 \text{ cm}$, the total power of terahertz emission in our laboratory experiments can reach 30–100 MW.

4. Conclusion

The spectral power of second harmonic electromagnetic emission, generated in the long-wavelength region of strong plasma turbulence due to coalescence of upper-hybrid waves, was calculated for different regimes of beam-plasma interaction, which can be realized in modern experiments at mirror traps. Computation results have shown that beam energy transferred to plasma oscillations in the regime of strong plasma turbulence can be converted efficiently to electromagnetic radiation. We have studied spectral characteristics of plasma emission in wide ranges of plasma density and turbulence energy and shown that the power of terahertz radiation in our beam-plasma experiments can reach tens of MW. Taking into account the angular distribution of this emission and possibility to change the emission frequency by varying the plasma density we come to the conclusion that terahertz radiation generated in a beam-driven strongly turbulent plasma can be very attractive for different applications.

Acknowledgments

Authors thank I.A. Kotelnikov for useful discussions. This work is supported by grant 11.G34.31.0033 of the Russian Federation Government, President grant NSh-5118.2012.2, Russian Ministry of Education and Science and RFBR grants 11-02-00563, 11-01-00249.

References

- [1] Gurnett D A, Anderson R R 1976 *Science* **194** 1159
- [2] Kruchina E N, Sagdeev R Z, Shapiro V D 1980 *JETP Lett.* **32** 419
- [3] Robinson P A, Cairns I H and Willes A J 1994 *Astrophys. J.* **422** 870
- [4] Mel'nik V N and Kontar E P 2003 *Sol. Phys.* **215** 335
- [5] Li B, Willes A J, Robinson P A and Cairns I H 2005 *Phys. Plasmas* **12** 012103
- [6] Gurnett D A, Shawhan S D and Shaw R R 1983 *J. Geophys. Res.* **88** 329
- [7] Cairns I H and Menietti J D 2001 *J. Geophys. Res.* **106**, 29515
- [8] Benford G, Tzach D, Kato K, Smith D F 1980 *Phys. Rev. Lett.* **45** 1182
- [9] Hopman H J and Janssen G C A M 1984 *Phys. Rev. Lett.* **52** 1613
- [10] Baranga A B, Benford G, Tzach D, Kato K 1985 *Phys. Rev. Lett.* **54** 1377
- [11] Arzhannikov A V, Burdakov A V, Kuznetsov S A, Makarov M A, Mekler K I, Postupaev V V, Rovenskikh A F, Sinitsky S L, Sklyarov V F 2011 *Fusion Sci. and Technol.* **59** (1T) 74
- [12] Vyacheslavov L N, Burmasov V S, Kandaurov I V, Kruglyakov E P, Meshkov O I and Sanin A L 1995 *Phys. Plasmas* **2** 2224
- [13] Vyacheslavov L N, Burmasov V S, Kandaurov I V, Kruglyakov E P, Meshkov O I, Popov S S and Sanin A L 2002 *Plasma Phys. Control. Fusion* **44** B279
- [14] Arzhannikov A V et al. 2003 *JETP Lett.* **77** 358
- [15] Arzhannikov A V, Burdakov A V, Koidan V S, Vyacheslavov L N 1982 *Physica Scripta* **T2**, 303
- [16] Timofeev I V, Lotov K V 2006 *Phys. Plasmas* **13** 062312
- [17] Timofeev I V 2012 *Phys. Plasmas* **19**, 044501
- [18] Willes A J and Melrose D B 1997 *Sol. Phys.* **171** 393
- [19] Kuznetsov A A 2007 *Plasma Phys. Rep.* **33** 482